Lab 4: Torsion Test

Author: **Alex Klein**
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Partners: Alan Huang
Class: 560.201
Section: 04
Group: A4
TA: Hamid Foroughi

Background and Objectives

Understanding how materials react to an applied load is essential for engineers to design structures that are both safe and effective. While there are many different loading conditions that must be analyzed, this lab focuses on the shear strain and angle of twist of an aluminum shaft when a torque is applied. With this information, and distance measurements taken of the setup, the shear modulus of the material can be calculated in two different ways. Both methods are used in the data analysis and each of their shear moduli are compared to published values. This also prompts a discussion into strain gauge placement, how torque and angle of twist are proportionally related, and the linearity of shear stress and shear strain in a shaft in torsion.

Experimental Methods and Analysis

A specially designed testing setup was used for the torsion test, as shown in figure (1) below. The system is comprised of a 2024-T4 aluminum shaft held on each end by one locked and one free support. Just inside the free support is a rigidly attached beam for application of a torque to the shaft, creating pure bending. On one side of the beam hangs a mount to add weight and apply a force due to gravity. On the other side, extending out to the right in the image below, a similar beam is contacted by a position transducer. Measuring the displacement of the end of the beam, we use trigonometry to calculate the angle of twist caused by the applied torque. Note that while both the length of the beam and arm extending from the central shaft are the same, these values could be any length and still used to calculate the torque applied and angle of twist.

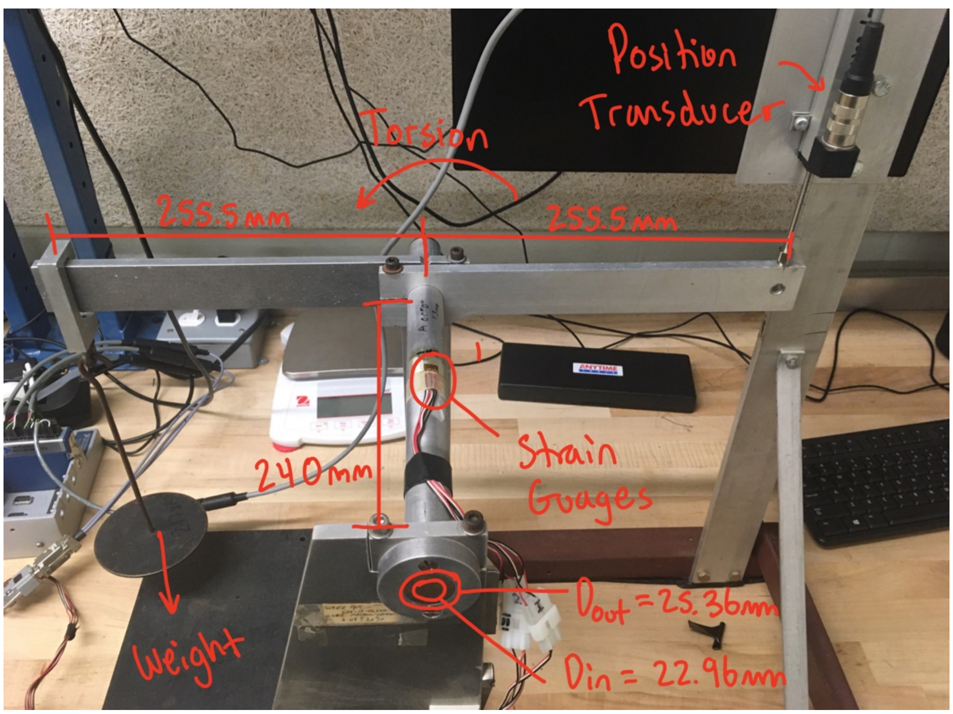

Figure (1): Diagram of Torsion Test Setup with Measurements

The image above displays the location of the strain gauges: in approximately the center of the torqued shaft. However, because we are measuring shear strain due to a shaft in torsion, the strain gauges are constructed in a special manner so their ability to measure normal strain is utilized to calculate shear strain. The strain gauge rosette is displayed in figure (2). With one strain gauge in line with the axis of the shaft, two more are offset 45 degrees to each side, so that the difference between the two perpendicular readings is the shear strain.

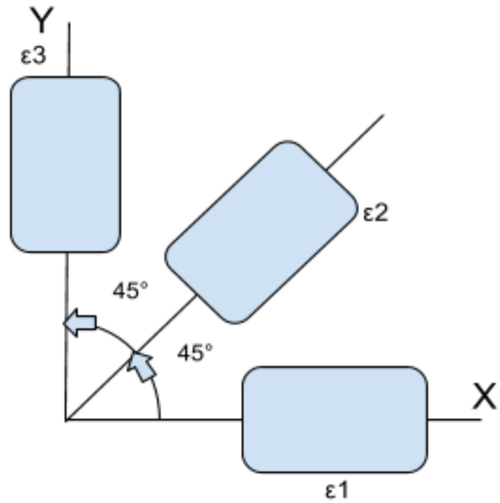

Figure (2): Diagram of Strain Gauge Rosette to Measure Shear Strain[1]

Before beginning the experiment, the position transducer was calibrated. With blocks stacked on top of each other, we recorded the position transducer's outputted voltage at various heights. The plot below displays the data measured and shows the regression line to the points. The slope and y-intercept were used to calibrate the position transducer in the LabVIEW computer software that runs the tension test program.

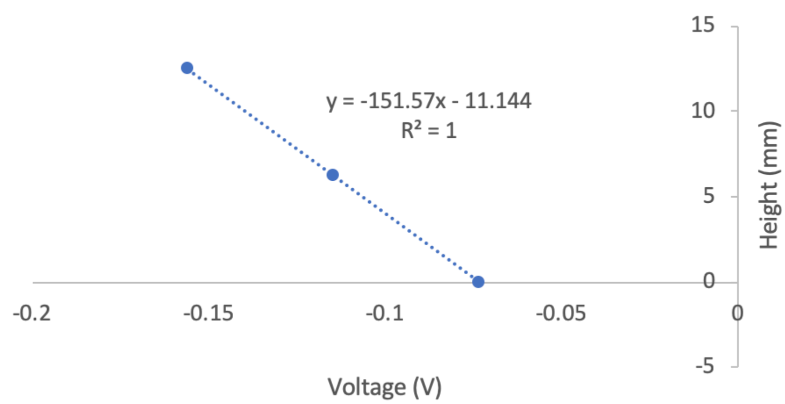

Figure (3): Regression Plot for Calibration of Position Transducer

The above plot reveals that for proper calibration, each voltage output (volts) from the position transducer should be multiplied by -151.57 and then deducted 11.144 in order to receive the calibrated height displacement (millimeters). The R^2 is 1, meaning the model is essentially a perfect fit.

[1]https://community.sw.siemens.com/servlet/rtaImage?eid=ka54O000000GzJ5&feoid=00N4O000006LZn5&refid=0EM4O00000112ar

After calibration, the experiment began in order to calculate the shear modulus. First, we measured the dimensions of the setup being used, as displayed in figure (1). Data was collected at four masses: 0.218kg, 0.718kg, 1.219kg, and 1.719kg hanging at the end of the beam where the force is meant to be applied. At each configuration, the calibrated strain data from each strain gauge and the distance traveled by the position transducer was recorded. The raw data recorded in the experiment is given in Appendix A. We then computed the shear modulus of the aluminum bar using two distinct methods.

Method 1 involves calculating the shear stress present in the bar with an applied torque and then calculating the shear modulus by using the shear strain. The shear stress τ of the shaft is defined as:

$$\tau = \frac{T\,r}{J} \tag{1}$$

Where r is the outer radius of the shaft, measured as 11.48mm. T is the torque applied to the shaft and J is its moment of inertia, both defined as:

$$J = \frac{\pi}{32}\left(D_{out}^4 - D_{in}^4\right) = 14620\ mm^4 \tag{2}$$

Where D_{out} is the outer diameter of the shaft and D_{in} is the inner diameter.

$$T = L_{arm} * F_{applied} \tag{3}$$

Where L_{arm} is the length of the moment arm that the weight hangs on and $F_{applied}$ is equal to the force of gravity of the mass applied at the distance.

Therefore, we can solve for the shear modulus G of the material. The following equation describes the relationship between shear stress and shear strain, which are both plotted to find the slope of the best fit regression line, equal to the modulus of rigidity:

$$G = \frac{\tau}{\gamma} \tag{4}$$

Where the shear strain γ is measured by the strain gauges, calculated by ϵ_1 and ϵ_3, the 45 degree offset strain gauges in the strain gauge rosette:

$$\gamma = \epsilon_1 - \epsilon_3 \tag{5}$$

Method 2 is performed by using the position transducer to calculate the angle of twist of the shaft to solve for G. The angle of twist θ is related to the distance traveled y by the position transducer:

$$\theta \approx \tan(\theta) = \frac{y}{L_{arm}} \tag{6}$$

Where G can be calculated with L_{shaft}, the length of the shaft:

$$G = \frac{L_{shaft} \cdot T}{\theta \cdot J} \tag{7}$$

After G is calculated for each mass applied on the shaft, the average value is determined from all trials.

Results

The table below displays the measured quantities of each trial, each value was calculated using equations (1), (3), (5), and (6) above.

Table (1): Measured Quantities During Experiment

Torque (Nmm)	ϵ1 (mϵ)	ϵ2 (mϵ)	ϵ3 (mϵ)	γ (mϵ)	y (mm)	θ (degrees)	τ (N/mm^2)
546	1.470	1.820	1.630	-0.160	0.104	0.000407	0.4776
1800	1.488	1.816	1.607	-0.119	0.297	0.001162	1.5731
3055	1.508	1.820	1.588	-0.080	0.489	0.001914	2.6708
4309	1.528	1.817	1.564	-0.036	0.702	0.002748	3.7663

The following plots help visualize the relationships between certain aspects of the shaft in torsion for discussion later in the report.

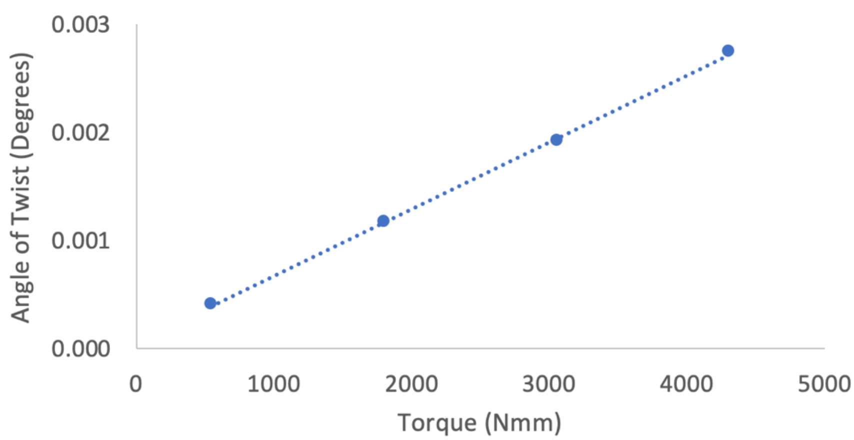

Figure (4): Torque vs. Angle of Twist

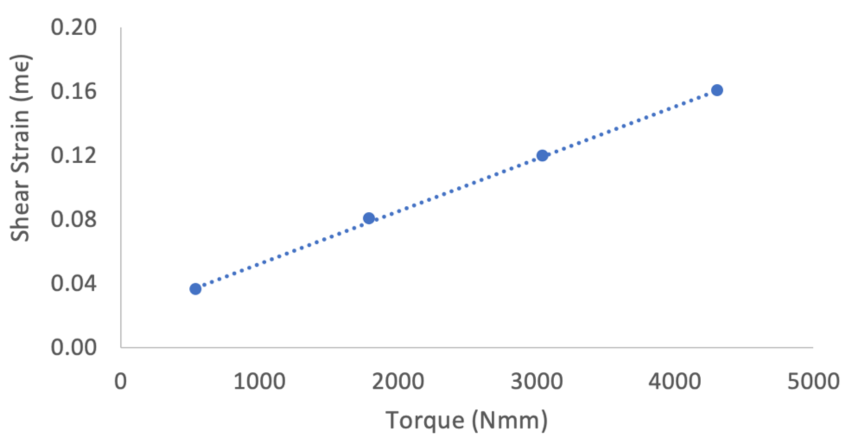

Figure (5): Torque vs. Shear Strain

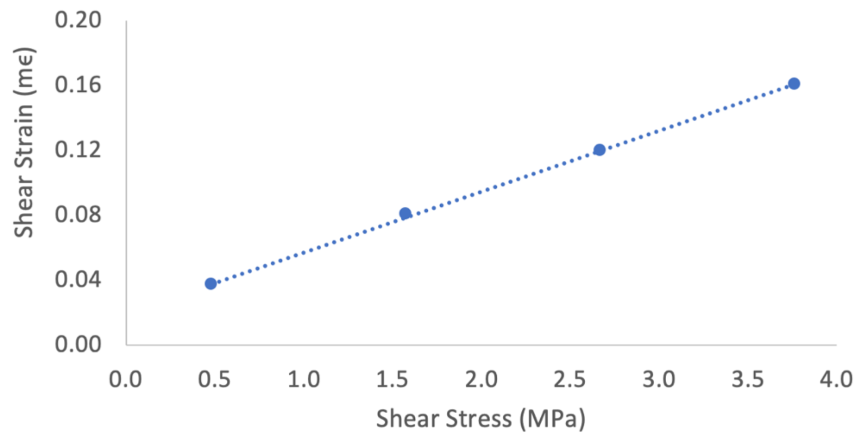

Figure (6): Shear Stress vs. Shear Strain

For Method 1, the modulus of rigidity G is calculated from the slope of figure (6) using the LINEST command in Microsoft Excel, with the results shown in table (2) below.

Table (2): Excel LINEST Results for G (Method 1)

	Value	Intercept
G (MPa)	26660.675	4.755
Uncertainty	450.185	0.049
R-Squared	0.999	0.041

With all calculations available, the following table provides an overview of the experiment results.

Table (3): Summary of Results

	Method 1	**Method 2**	**Published Value**
Value for G (GPa)	26.7	24.9	28.0
Percent Error (%)	4.6	11.1	-

Discussion

The experiment predicted the modulus of rigidity moderately accurately. The two methods of calculation had percent errors of 4.6% and 11.1% off of the published 28 GPa value. While the results are not perfect and probe questions into lab methods due to greater than 10% error on one of the models, the experiment correctly identifies the modulus of rigidity at a relatively accurate level considering its order of magnitude. There are no outliers in the data and no major errors occurred or noticed in the lab, therefore making the authors confident in the results of the experiment. Additionally, all linear trends in the lab follow near perfect (R^2 = 1) relationships found in the regression analysis.

The lab focused on two separate methods to determine the shear modulus of 2024-T4 aluminum. The two methods performed generally similar on the modulus of rigidity's scale, but relative to each other, method 1 performed better than method 2.

The first method calculated the shear stress applied to the material and measured the shear strain with strain gauges, then found the slope of the linear regression line of the two plotted values, equal to the shear modulus. At small torques, the angle of twist would be negligible in affecting the results. Therefore, shear stress and strain should be linearly proportional throughout the experiment. This can be seen in figure (6) in the results section and quantitatively analyzed by the regression results in table (2). The relationship between shear stress and strain we very highly linear, with an almost perfect goodness of fit value R^2 = 0.999. Such linearity is utilized by method 1, which computes G from the slope of the two plotted values. This method also saw a very small uncertainty in the resulting calculation, only off of the published shear modulus by a 4.6% error.

The second method used a position transducer to measure the angle of twist of the torqued shaft, which the shear modulus was calculated from. This method as not as accurate, with an 11.1% error. A possible reason for the larger percent error, in addition to the sources of error discussed later, is from the arrangement of the strain gauges. While the devices are designed to only measure normal strain (which is something that would occur due to axial loading), they can be specially configured into strain rosette to measure both normal and shear strain. In this method, one strain gauge is aligned along the axis of the shaft while the others are each 45 degrees offset in angle to that center strain gauge on either side. Therefore, the difference between the two angled ones is equal to the shear strain ($\gamma = \epsilon_1 - \epsilon_3$). The table in Appendix A displays the raw data recorded from the strain gauges, where it is evident that outer strain gauge readers grow further apart with an increased torque. This makes sense: a larger applied load causes a larger shear strain. While these vary, the center strain gauge (ϵ_2) remains relatively constant at approximately 0.0018 mm/mm. This also makes sense, as the strain gauge in line with the axis only records axial loads, of which none are placed on the shaft. Error could arise in this part of the lab if any of the strain gauges were not placed at perfect 45-degree angles.

The error from the experiment arises in the sensors, procedure, and human completion. The sensors used in the experiment inherently have error associated with them. The position transducer was calibrated before the start of the experiment, but we noticed that during the experiment, with no weight present in the setup, the voltage reading and height reading did not perfectly read zero. Instead, we possibly should have redone the calibration due to any slight errors made during the first process, which would reduce error in method 2 of the experiment. Additionally, our lab manager had previously noticed problems with the strain gauges used in the lab, contributing error to method 1.

The setup itself, specifically the support system for the beam and weights, was not perfectly ideal. During the calculation and analysis process, we ignored any friction that may have taken place in the supports (on the free end of the shaft), any possible bending the beam which applies torque, and the change in the angle of force applied that is transferred into torque, to name a few. All of these phenomena, however, may have contributed to the difference between the recorded values and the published one in both methods.

Finally, we assumed that the length of the shaft was a single value, but the couplings to the beam which applied torque and to the arm measured by the position transducer are at slightly different locations on the shaft. Therefore, the angle of twist at each length is slightly different (larger for the applied torque than the positions transducer measurement), negatively affecting the accuracy in method 2. Instead, both arms should have been one connected at the same point or simply one single piece. Additionally, if the contacts with the shaft were more concentrated, they would have acted more like point forces, improving the accuracy of the measurements and calculations.

Conclusion

The goal of this experiment was to calculate the shear modulus of a 2024-T4 aluminum shaft in a torsion test. This was completed through two different methods measuring different values associated with an applied torque on the shaft. First, the modulus of rigidity was derived from the slope of the linear relationship between shear stress and strain, resulting in 26.7 GPa. The second method measured the angle of twist of the shaft with a position transducer to calculate G as 24.9 GPa. Each method had a 4.6% and 11.1% error off from the published 28 GPa value, respectively. The authors of the report and experiment are confident in the results, despite moderately large percent errors, due to no noticeable major problems in the lab and the highly linear fits of all the data. Accurate calculations of shear modulus of materials are an essential task, providing valuable information for the design of devices and structures which require the proper load analysis for maximum safety and performance.

Appendix

Appendix A: Raw Data Table of Results

Trial	Mass (kg)	Force (N)	Torque (Nmm)	ϵ1 (mϵ)	ϵ2 (mϵ)	ϵ3 (mϵ)	y (mm)
1	0.218	2.138	546	1.47	1.82	1.63	0.104
2	0.718	7.043	1800	1.488	1.816	1.607	0.297
3	1.219	11.958	3055	1.508	1.82	1.588	0.489
4	1.719	16.863	4309	1.528	1.817	1.564	0.702