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Lab 10: Diffraction

Experiment Purpose

In this lab, we utilized the principles of the diffraction of light to experimentally determine (1) the wavelength of a laser pointer, (2) the spacing between Z-bands in muscle tissue, and (3) analyze the structure of a butterfly's compound eye.

Calculations and Error Propagation

Recall the equation that describes the inference pattern:

$$dsin\theta_n = n\lambda$$

Where d is the distance between the lines of the grating, λ is the wavelength of the incoming light, θ_n is the angle between the direct light path and the n order of light projected onto the screen.

Solving for $sin\theta_n$, we find the following equation in the form $y = mx + b$, where the slope $\left(\frac{\lambda}{d}\right)$ can be determined by plotting $sin\theta_n$ on the y-axis vs. n on the x-axis.

$$sin\theta_n = \left(\frac{\lambda}{d}\right)n$$

We use the Pythagorean theorem to solve for d, and then solve for $sin\theta_n$ in terms of x (the distance between the grating and the screen) and y_n (the distance from the direct path of light to the n order.

$$d = \sqrt{x^2 + {y_n}^2}$$

$$sin\theta_n = \frac{y_n}{\sqrt{x^2 + {y_n}^2}}$$

In the three sections of the lab, we use the LINEST function in Excel to find the linear regression of our recorded data. Certain parameters are found in each part, and another variable in the above equations is subsequently calculated from that.

The following equations allow for proper error propagation in the measurement and data analysis process. The variable m is the slope of the linear best fit line to the data.

$$\delta\lambda = \sqrt{\left(\frac{\delta d}{m}\right)^2 + \left(\frac{\delta m}{m^2} \cdot d\right)^2}$$

$$\delta d = \sqrt{(\frac{\delta\lambda}{m})^2 + (\frac{1}{m^2}\delta m \cdot \lambda)^2}$$

$$\delta(sin\theta_n) = \left(\frac{x}{(x^2 + {y_n}^2)^{3/2}}\right)\sqrt{(x \cdot \delta y_n)^2 + (y_n \cdot \delta x)^2}$$

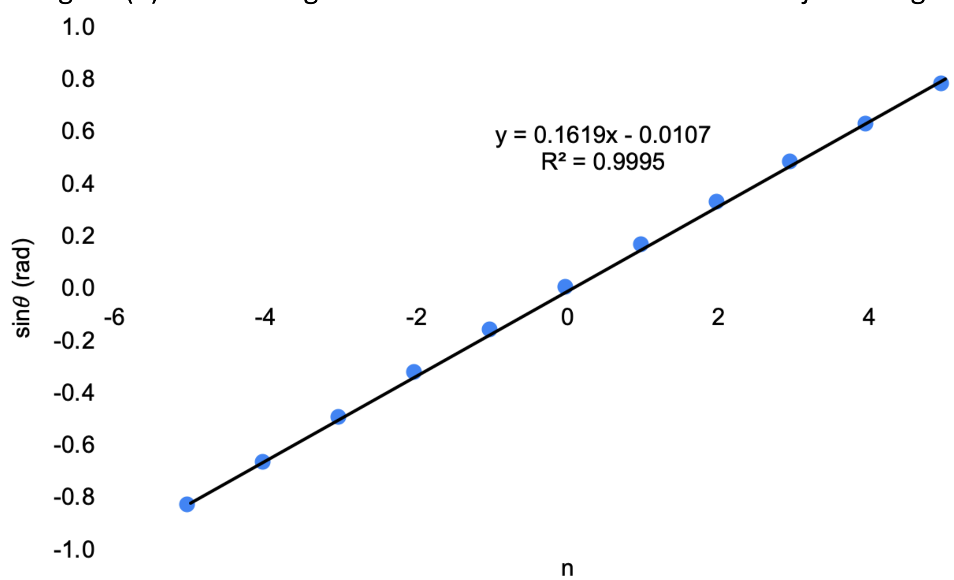
Data Analysis

Part 1: Pointing the laser through a known 300lines/mm grate, we calculated the wavelength of the laser. The true wavelength, as noted on the device itself is $\lambda = 532nm$.

Table (1): LINEST Results for Angle Calculated to Projected Light

	Slope (mm^2/lines)	Intercept (rad)
Value	0.16188	-0.01073
Uncertainty	0.00121	0.00381
R-Squared	0.99950	0.01265

Figure (1): Plot of Angle Calculated at Several Orders of Projected Light

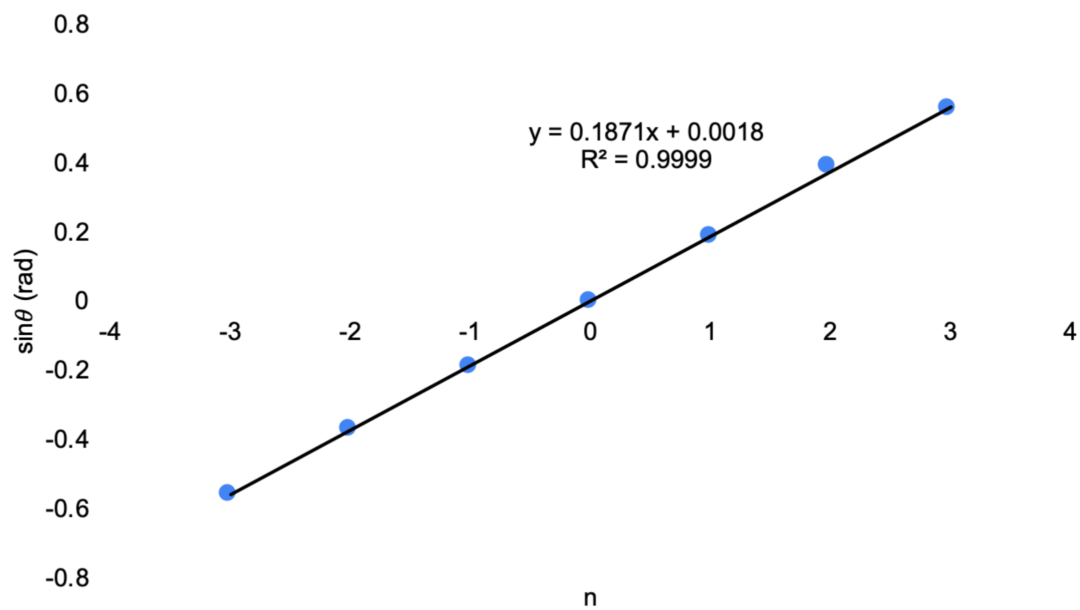


Part 2: Pointing the same laser at a muscle sample, we used the equations mentioned in the last section to calculate the distance between the Z-band grating structure of the biological specimen.

Table (2): LINEST Results for Angle Calculated to Projected Light

	Slope (mm^2/lines)	Intercept (rad)
Value	0.18707	0.001796
Uncertainty	0.00098	0.001965
R-Squared	0.99986	0.005201

Figure (2): Plot of Angle Calculated at Several Orders of Projected Light

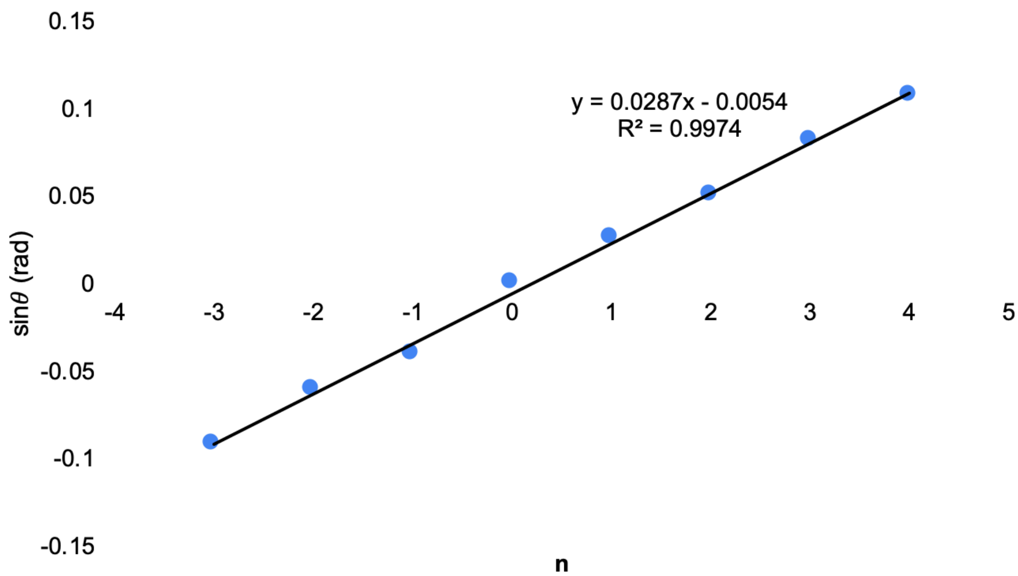


Part 3: After removing an eye of a butterfly and properly preparing the nerve tissue on a slide, we used the principles of diffraction explored in the previous parts to calculate the dimensions of the hexagonal ommatidia.

Table (3): LINEST Results for Angle Calculated to Projected Light

	Slope (mm^2/lines)	Intercept (rad)
Value	0.02870	-0.00543
Uncertainty	0.00060	0.00141
R-Squared	0.99736	0.00390

Figure (3): Plot of Angle Calculated at Several Orders of Projected Light



Results

Part 1: wavelength of laser pointer light $\boldsymbol{\lambda = 534 \pm 5\ nm}$
Part 2: distance between the Z-band grating structure $\boldsymbol{d = 2.84 \pm 0.09\ \mu m}$
Part 3: distance between vertices of hexagonal ommatidia $\boldsymbol{d = 18.5 \pm 0.4\ \mu m}$

Discussion: Observations

- All three parts of the lab displayed positive linear relationships between the order of maxima of the light diffracted onto the screen and $sin\theta_n$ of the associated angle between the order and direct path of light – this makes sense from the equations previously discussed and was expected
- In part 1, using a manufactured grate, the light was diffracted in clear orders n = -2, -1, 0, 1, etc. while in parts 2 and 3 of the lab, using imperfect biological specimens, the diffracted light was not as crisp and clear
- The experiment setup allowed us to properly measure distances between the laser/grate/screen and between diffracted light points as described in the lab handout
- In parts 1 and 2, we held a ruler up to the screen to measure the distances y_n, while in part 3 we drew a dot on the screen with pencil at each location because the high concentration of diffracted light made it difficult to keep track of the measurements

Discussion: Significance

- Part 1: the actual wavelength of the laser used was 532 nm, compared to the $534 \pm 5\ nm$ value we received: 0.4% error; there was an associated goodness of fit value of the linear regression $R^2 = 0.9995$
- Part 2: we measured the distance between the Z-bands in the muscle tissue to be $2.84 \pm 0.09\ \mu m$ apart; compared to the expected value of 2.05 μm to 2.60 μm is a 9.2% error (at the smallest difference from the expected); there was an associated goodness of fit value of the linear regression $R^2 = 0.9999$
- Part 3: we measured the distance between vertices of hexagonal ommatidia to be $18.5 \pm 0.4\ \mu m$; compared to the expected value of approximately 30 μm, there is a 38% error; there was an associated goodness of fit value of the linear regression $R^2 = 0.9974$

Discussion: Confidence

- We are very confident in the results of the experiment due to the (1) relatively low percent errors, (2) high goodness of fit values, and (3) little error observed during the lab
- (1) Part 1 of the lab displayed very small error (0.4%), while parts 2 and 3 were slightly higher (9.2% and 38% respectively); this makes sense because the grates used were from biological specimens, so each one is expected to vary slightly from another grate that was used to calculate the expected value; we were correctly "on the order" of the expected value of the distances between the Z-bands and vertices in parts 2 and 3
- (2) All data collected was linearly regressed using the LINEST function in Excel, for which each line of best fit line had a value greater than $R^2 = 0.99$, very high goodness of fit – showing that the model properly describes the data
- (3) There were no major errors observed in the lab, however several small flaws of the experiment's procedure were noticed that may have affected both the accuracy and precision of the results:
  - The wide screen was mounted on the track by a single pivot, which allowed it to accidentally rotate; this may have affected our measurements for y_n, and therefore $sin\theta_n$, because the angle between the screen and light ray was not exactly 90 degrees
  - As mentioned before, the light diffracted on the screens in parts 2 and 3 was blurry and fuzzy, so we measured the distance from the center points of each dot, which was not a precise measurement